Cooperative Ramp Merging for Mixed Traffic with Connected Automated Vehicles and Human-Operated Vehicles

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Cooperative Ramp Merging

Hybrid Model Predictive Control and Real Time Computations

Microscopic Right-of-Way Trading Mechanism

Modeling Car-Following Heterogeneities

Short-term Trajectory Prediction

Trajectory Planning and Tracking
Traffic conflict is the most fundamental problem in transportation science and engineering.

At least in theory, it is possible to mitigate or eliminate traffic conflicts, in the mixed traffic environment with connected automated vehicles (CAVs) and human-operated vehicles (HV)

The proposed mechanism is called Cooperative Decision-Making for Mixed Traffic (i.e., CDMMT)
Background

China, EU and US will start to make highly automated cars around 2020-2025

DA/PA → CA/HA → FA

Information exchange → sensing and fusion → **cooperative decision-making and control**

Source: **Chinese National Standard for CAV Industry**
Key Challenges

Technical difficulties *(hardware, sensing, communication)*

Mixed traffic *(human are myopic, stochastic, and non-cooperative)*

System-efficient *(cooperation is not necessarily system-improving)*

Ethical dilemma *(puppy vs. a group of people)*

GM's Autonomous Car Gets Confused, Stops for Lunch

While test driving GM's latest autonomous vehicles, some passengers found themselves stuck in traffic.

Apple self-driving test car gets rear-ended by a Nissan Leaf in first ever crash

*Predictably, it was a human's fault*

By Nick Statt | @nickstall | Aug 31, 2016, 7:18pm EDT
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Trajectory Planning and Tracking
CDMMT: A Bi-level Programming Framework

upper-level: merge sequencing
lower-level: trajectory design

**Upper-level:** merge sequencing (filling $n$ ramp vehicles into $m+1$ gaps)

$$f_k(s_k) = \min_{x_k \in x_k, \hat{q}_k^k = \{q_1^k, q_2^k, ..., q_T^k\}} \{D_k(s_k, x_k) + f_{k-1}(s_{k-1})\}, k = 1, 2, ..., n$$

s.t. $f_0(s_0) = 0$

$s_1 = m + 1$

$s_{k+1} = s_k - x_k + 1, k = 1, 2, ..., n - 1$

$$D_k(s_k, x_k) = g_{t_f}^k \left(p_{t_f}^k\right) + \hat{g}^{k-1}$$

$f_k(s_k)$: The minimum system cost from the initial stage to stage $k$

$s_k$: The number of available mainline gaps for ramp vehicle $k$

$x_k$: The gap taken by ramp vehicle $k$

$D_k$: The cost of the merge maneuver pertaining to ramp vehicle $k$

$g_{t_f}^k$: The objective function of the lower-level trajectory design problem pertaining to ramp vehicle $k$

$\hat{g}^{k-1}$: The cost for mainline vehicles that are not directly involved in any merging maneuver
### Case-based control strategies

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Cooperative Merging Control Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1: HHA</td>
<td>R2/R3: vehicle 3 slower</td>
</tr>
<tr>
<td>C2: HAA</td>
<td>R1/R4: vehicle 2 slower, vehicle 3 slower; R2: vehicle 2 faster, vehicle 3 slower; R3: vehicle 2 undetermined, vehicle 3 slower</td>
</tr>
<tr>
<td>C3: AHA</td>
<td>R1: Vehicle 1 faster; R2: Vehicle 3 slower; R3/R4: Vehicle 1 faster &amp; Vehicle 3 slower</td>
</tr>
<tr>
<td>C4: AAA</td>
<td>R1: Vehicle 1 faster, Vehicle 2 slower, Vehicle 3 slower; R2: Vehicle 2 faster, Vehicle 3 slower, Vehicle 1 faster; R3/R4: Vehicle 1 faster, Vehicle 2 undetermined, Vehicle 3 slower</td>
</tr>
<tr>
<td>C5: HAH</td>
<td>R1: Vehicle 2 slower; R2: Vehicle 2 faster; R3: Vehicle 2 undetermined;</td>
</tr>
<tr>
<td>C6: AHH</td>
<td>R1/R3: Vehicle 1 faster</td>
</tr>
<tr>
<td>C7: AAh</td>
<td>R1: Vehicle 1 faster, Vehicle 2 slower; R2: Vehicle 1 faster, Vehicle 2 faster; R3: Vehicle 1 faster, Vehicle 2 slower</td>
</tr>
<tr>
<td>C8: AHN</td>
<td>R1/R3: Vehicle 1 faster</td>
</tr>
<tr>
<td>C9: AAN</td>
<td>R1/R3: Vehicle 1 faster, Vehicle 2 slower</td>
</tr>
<tr>
<td>C10: HAN</td>
<td>R1/R3: Vehicle 2 slower</td>
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<tr>
<td>C11: NHA</td>
<td>R2/R3: Vehicle 3 slower</td>
</tr>
<tr>
<td>C12: NAA</td>
<td>R2/R3: Vehicle 2 faster, Vehicle 3 slower</td>
</tr>
<tr>
<td>C13: NAH</td>
<td>R2/R3: Vehicle 2 faster</td>
</tr>
</tbody>
</table>

“N” stands for “Null” and it is used as a placeholder. “H” stands for a human-operated vehicle; “A” stands for a CAV.

R 1: vehicle 2 is “too close” to vehicle 1
R 2: vehicle 2 is “too close” to vehicle 3
R 3: vehicle 2 is neither “too close” to vehicle 1 nor vehicle 3, but the it is uncomfortable to merge
R 4: vehicle 2 is “too close” to vehicle 1 and vehicle 3
**Lower-level:** trajectory design (multi-object optimal control)

\[
g_t^k(p_t^k) = \begin{cases} 
\min_{q_t^k \in Q_t^k} \{d_t^k(p_t^k, q_t^k) + g_t^k(p_t^k)\}, & q_t^k \neq \emptyset \\
\infty, & q_t^k = \emptyset
\end{cases}, \quad \forall \tau = t_0^k + \tau, t_0^k + 2\tau, ..., t_f^k, \ k = 1, 2, ..., n
\]

\[
g_{t_0}^k(p_{t_0}^k) = \sum_{i \in K} (v_i(t_0^k) - v^e)^2, \ \forall k = 1, 2, ..., n
\]

\[
p_{t_0}^k = \{v_i(t_0^k), l_i(t_0^k) | i \in K\}, \ \forall k = 1, 2, ..., n
\]

**State transition:**

\[
l_i(t + \tau) = l_i(t) - v_i(t)\tau - 0.5u_i(t)\tau^2,
\]

\[
\forall i \in K, t = t_0^k, t_0^k + \tau, ..., t_f^k - \tau, k = 1, 2, ..., n
\]

\[
v_i(t + \tau) = v_i(t) + u_i(t)\tau,
\]

\[
\forall i \in K, t = t_0^k, ..., t_f^k - \tau, k = 1, 2, ..., n
\]

**Final-state constraint:**

\[
U_k(t_f^k) \geq 0, \forall k = 1, 2, ..., n
\]

---

- \(t_0^k, ..., t_f^k\): The time stamps of a trajectory design period
- \(p_t^k\): The states of all vehicles in set \(K\) at time \(t\)
- \(q_t^k\): The set of decisions of vehicles in set \(K\) at time \(t\)
- \(v^e\): Desired speed
- \(U_k(t_f^k)\): Merging utility
Lower-level: trajectory design (multi-object optimal control)

Vehicle $k$

Non-cooperative:

$$u_k(t + 1) = u_{mic}(S_k(t), v_k(t), v_l(t))$$

Faster:

$$v_k(t) + u_k(t)\tau \geq v_{mic}(S_k(t), v_k(t), v_l(t))$$
$$v_k(t) + u_k(t)\tau \leq v^e$$

Slower:

$$v_k(t) + u_k(t)\tau \leq v_{mic}(S_k(t), v_k(t), v_l(t))$$
$$v_k(t) + u_k(t)\tau \geq 0$$

Undetermined:

$$v_k(t) + u_k(t)\tau \leq v^e$$
$$v_k(t) + u_k(t)\tau \geq 0$$

Longitudinal dynamics

Vehicle $\hat{K}^{lead}$

Non-cooperative:

$$u_{\hat{K}^{lead}}(t + 1) = u_{mic}(S_{\hat{K}^{lead}}(t), v_{\hat{K}^{lead}}(t), v_l(t))$$

Faster:

$$v_{\hat{K}^{lead}}(t) + u_{\hat{K}^{lead}}(t)\tau \geq v_{mic}(S_{\hat{K}^{lead}}(t), v_{\hat{K}^{lead}}(t), v_l(t))$$
$$v_{\hat{K}^{lead}}(t) + u_{\hat{K}^{lead}}(t)\tau \leq v^e$$

Vehicle $\hat{K}^{fol}$

Non-cooperative:

$$u_{\hat{K}^{fol}}(t + 1) = u_{mic}(S_{\hat{K}^{fol}}(t), v_{\hat{K}^{fol}}(t), v_l(t))$$

Slower:

$$v_{\hat{K}^{fol}}(t) + u_{\hat{K}^{fol}}(t)\tau \geq 0$$
$$v_{\hat{K}^{fol}}(t) + u_{\hat{K}^{fol}}(t)\tau \leq v_{mic}(S_{\hat{K}^{fol}}(t), v_{\hat{K}^{fol}}(t), v_l(t))$$
Lateral dynamics

Longitudinal dynamics

- Newell’s simplified car following model
- Gipps
- IDM/EIDM
- ......
Results – q1:1000veh/h   q2:1000veh/h   50% CAV

Speed profiles and acceleration profiles

non-cooperative

cooperative

Speed contours

non-cooperative

cooperative
• Capacity increases with the increase of CAV penetration (up to about 20%)
• CDMMT can further improve the capacity by about 10% - 15% in the case of high penetration

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System Stochasticity is not considered in the deterministic-CDMMT mechanism.

The centralized control improves system-efficiency, however, computational efficiency becomes a critical issue.

Solution:

- Closed-loop control (model predictive control)
- Hybrid Centralized-Decentralized system
- Use computational-efficient solution approach

**Step 1.** At time $t$, the communication between the RSU and the CAVs in the communication zone is established. Then the states of vehicles will be adopted and shared by RSU (centralized controller).

**Step 2.** As shown in the Figure(b), after the establishment of communication, the centralized controller will assign the specific lower-level problem computing tasks to OBU through the iteration of upper-level problem. All the OBU controllers will receive the assignment of the lower-level problem and the information of neighboured vehicles.

**Step 3.** OBU solve the lower-level problem and return the optimal trajectory consisting of speed and location to the centralized controller.

**Step 4.** RSU solves the upper-level sequencing problem according to the feedback in Step3 and transfers the optimized merging sequences to CAVs. Just as shown in Fig.(c), ramp CAV determines the specific merging gap and implements the lower-level longitudinal control after receiving the order of RSU. Then go to Step 1 ($t = t+\tau$).
\[ \exists T_c^k, \ l_k(t_0 + T_c^k + \tau |t_0) \geq l_e \land l_k(t_0 + T_c^k |t_0) \leq l_e) \cdot l_k(t_0 + T_c^k |t_0) \]

\[ l_k(t_0 + T_c^k |t_0) \] denotes the location of vehicle \( k \);

\[ (t_0 + T_c^k |t_0) \] represents the predictive time \( t_0 + T_c^k \) at real time \( t_0 \);

\[ T_c^{k,lead} = T_c^{k,fol} = T_c^k \]

\[ T_p = T_c^n \]

\( l_e \): the end of ramp control zone
Results – Flow-Density Diagrams

(A) 10% CAV

(B) 30% CAV

(C) 60% CAV

(D) 80% CAV

- Non-cooperative
- Open-loop
- MPC
# Results – Average Computational Time (Seconds)

<table>
<thead>
<tr>
<th>Pene</th>
<th>Average Lower-level optimization cases per timestamp</th>
<th>Calculation time of Centralized system (Seconds)</th>
<th>Calculation time of hybrid system (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DMC</td>
<td>DP</td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td>1.03</td>
<td>0.056</td>
</tr>
<tr>
<td>40%</td>
<td></td>
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<tr>
<td>60%</td>
<td></td>
<td>1.61</td>
<td>0.095</td>
</tr>
<tr>
<td>80%</td>
<td></td>
<td>1.56</td>
<td>0.11</td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td>1.76</td>
<td>0.11</td>
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</table>

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<td></td>
<td></td>
<td>DMC</td>
<td>DP</td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td>1.89</td>
<td>0.11</td>
</tr>
<tr>
<td>40%</td>
<td></td>
<td>2.73</td>
<td>0.16</td>
</tr>
<tr>
<td>60%</td>
<td></td>
<td>3.41</td>
<td>0.20</td>
</tr>
<tr>
<td>80%</td>
<td></td>
<td>3.67</td>
<td>0.24</td>
</tr>
<tr>
<td>100%</td>
<td></td>
<td>3.69</td>
<td>0.24</td>
</tr>
</tbody>
</table>

1. Cooperative Ramp Merging
2. Hybrid Model Predictive Control and Real Time Computations
3. Microscopic Right-of-Way Trading Mechanism
4. Modeling Car-Following Heterogeneities
5. Short-term Trajectory Prediction
6. Trajectory Planning and Tracking
In academia, CAVs are always designed to be more cooperative, which conflicts the self-interest nature of human.

Under the assumption of rationality, both CAVs and HVs can **behave cooperatively** (i.e., yielding or slowing down) if enough incentive can be provided.

The proposed mechanism is called Microscopic Right-of-Way Trading Mechanism for Cooperative Decision-Making (i.e., Micro-ROWTM).

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Microscopic Right-of-Way Trading Mechanism

- Right-of-way trading
- Mixed traffic (NCVs & CVs)
- Game-theoretic
- Individual rationality & system-efficiency
- Dominant-strategy incentive-compatibility (DSIC) under incomplete information
- Envy-minimization
Definitions

Total cost of payer (A)/receiver (B): $\Delta U_j = \eta_j \Delta e_j + \sigma_j (\hat{t}_j^f - \hat{t}_j^0 - t_j^f + t_j^0) + \kappa_j \left( \sum_{t=\hat{t}_j^f}^{t_f} g_j(t) - \sum_{t=\hat{t}_j^0}^{t_0} g_j(t) \right)$

Payoff of payer: $N_A \triangleq \Delta U_A - p = r_A \times \Delta c_A^T - p$

Payoff of receiver: $N_B \triangleq \Delta U_B + p = r_B \times \Delta c_B^T + p$

Total avenue: $\Delta \omega \triangleq N_A + N_B = r_A \times \Delta c_A^T + r_B \times \Delta c_B^T$

Trading rules

1. Equal Allocation:
   The payoff of payer and receiver is equal (i.e., half of the total revenue).

2. Double auction:
   The trading price is a linear combination of the payoffs reported by payer and receiver.

3. Dynamic Negotiation:
   Imitating the process of bargaining in reality to determine a trading price that both payer and receiver are satisfied with.

4. A constrained optimization method:
   Using the optimization method to find a trading price meets the conditions of individual rationality, system-improvement, and DSIC to minimize the envy.
Value of time: **80 RMB/hour**
Fuel cost: **7 RMB/Liter**
Applying CDMMT Ramp Merging on congested ramps, the direct cost saving is around **80-100 RMB/hour/section**
Applying CDMMT Ramp Merging on the 47 merge section of the 2nd ring road of Chengdu, the direct cost saving is around **7M RMB per year**

The travel time and fuel consumption saved by each group optimization under different flow ratios. ($q_1$: mainline flow, $q_2$: ramp flow)

$q_1: q_2 = 900:900$ (light traffic): Each group saved **2.29 seconds** and **35.65g fuel** on average;
$q_1: q_2 = 1300:1000$ (heavy traffic): Each group saved **1.96 seconds** and **38.55g fuel** on average;

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According to Litman’s prediction, by 2040, 50% of traffic will be CAVs (Litman, T., 2014).

The emergence of connected automated vehicles (CAVs) has led to the problem of mixed traffic, i.e., traffic comprised of conventional human-operated vehicles (HVs) and CAVs (Huang, T., and Z. Sun., 2019).

In mixed traffic, the decision-making and/or control of CAVs largely depends on accurate description and prediction of HVs’ behaviors (Chen, D. et al., 2020; Jin, S. et al., 2020).

This underscored the necessity of better understanding the heterogeneities of human driving behaviors.

Model Calibration results
• Some parameters are highly consistent across the board, while others are quite different in different leader-follower compositions

PCA and FCM results
• The orders of Weighted contribution of feature \( (WCF) \) are quite different across different compositions.
• Such clustering results were attributed by the underlying driving style differences.
• The results are also consistent with the commonly recognized “aggressive-normal-mild” driving style classification.
Distribution fitting for car-following parameters
- Stable distribution has the best performance compared to the other distributions

Comparisons between CCF and UCF models
- In all cases, the estimation errors of CCF models are much smaller compared to UCF model, clearly show that the proposed CCF models can more accurately describe the heterogeneities in car-following behaviors.

Transferability analysis using US-101 dataset
- The CCF models calibrated using FVD and IDM in general outperforms the GHR model.
- The estimation errors of CCF models are much smaller compared to UCF model
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Short-term Trajectory Prediction

Trajectory Planning and Tracking
Accurately predicting trajectory of surrounding manual vehicles is the key premise to ensure that the CAVs can plan its own trajectory safely and reliably.

**Learning-based LSTM network (GR-LSTM)**

Short-term Trajectory Prediction

Architecture of GR-LSTM model
Short-term Trajectory Prediction

Where: $X_{v_i} = [x_{v_i}^{t_0-T_{obs}}, x_{v_i}^{t_0-T_{obs}+\tau}, \ldots, x_{v_i}^{t_0}]$, $\forall v_i \in V$, where $X_{v_k}^{t_0} = (x_{v_k}^{t_0}, y_{v_k}^{t_0})$

$h_{v_j}^{m-1} = LSTM(X_{v_i}, h_0) \ t \in \{t_0 + 1 \ldots t_0 + T_{pred}\}$

$h_N^{k} (v_i) = Aggregator(h_{v_j}^{k-1}, h_{v_i}^{k-1}) \ \forall v_j \in N (v_i), k = 1,2 \ldots m - 1$

$h_{v_i}^{k} = \sigma (W^k \cdot \text{concatenate}(h_{v_i}^{k-1}, h_N^{k} (v_i))), k = 1,2 \ldots m - 1$

$h_{v_j}^{m,t-1} = Aggregator(h_{v_j}^{m-1}, s_{t-1}) \ \forall v_j \in N (v_s), t \in \{t_0 + 1 \ldots t_0 + T_{pred}\}$

$C_t = \sigma (W \cdot \text{concatenate}(h_{v_s}^{m-1}, h_{v_s}^{m,t-1}))$

$s_t = LSTM(s_{t-1}, C_t), t \in \{t_0 + 1 \ldots t_0 + T_{pred}\}$

$\hat{y}_t = MLP(s_t; W_z)$
### Results

**Baselines:**

*Vanilla-LSTM (V-LSTM)[1]*: uses a sequence of past trajectories to predict a sequence of future trajectories

*Social LSTM (S-LSTM)[2]*: model of an LSTM-based neural network with social pooling for pedestrian trajectory prediction

*Interaction-aware Kalman neural network (IaKNN)[3]*: added a Kalman filter layer to the interaction-aware layer

*Convolutional social pooling LSTM (CS-LSTM)*: LSTM with convolutional social pooling and maneuvers, including the maneuver-based decoder used for generating a multimodal predictive distribution

#### Quantitative Results:

**Quantitative results of our GR-LSTM compared with those of baseline approaches. Evaluation metrics are reported in terms of RMSE in meters.**

<table>
<thead>
<tr>
<th>Prediction horizon(s)</th>
<th>V-LSTM</th>
<th>S-LSTM</th>
<th>IaKNN</th>
<th>CS-LSTM</th>
<th>GR-LSTM(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.74</td>
<td>0.68</td>
<td>0.62</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>2</td>
<td>1.44</td>
<td>1.28</td>
<td>1.03</td>
<td>1.27</td>
<td>1.17</td>
</tr>
<tr>
<td>3</td>
<td>2.57</td>
<td>2.27</td>
<td>1.97</td>
<td>2.09</td>
<td>1.74</td>
</tr>
<tr>
<td>4</td>
<td>4.23</td>
<td>3.32</td>
<td>2.93</td>
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<td>5</td>
<td>5.92</td>
<td>4.46</td>
<td>4.12</td>
<td>4.27</td>
<td>3.32</td>
</tr>
</tbody>
</table>


Results

Relationship Between The Number Of Rows Of Forward Vehicles and The Prediction Accuracy

Quantitative results of the selection of different rows of forward vehicles in the GR-LSTM model. Each cell in the table is the RMSE/ADE.

<table>
<thead>
<tr>
<th>Prediction horizon(s)</th>
<th>GR-LSTM(0)</th>
<th>GR-LSTM(1)</th>
<th>GR-LSTM(2)</th>
<th>GR-LSTM(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.69/0.47</td>
<td>0.68/0.46</td>
<td>0.68/0.47</td>
<td>0.70/0.48</td>
</tr>
<tr>
<td>2</td>
<td>1.35/0.79</td>
<td>1.22/0.73</td>
<td>1.17/0.70</td>
<td>1.28/0.75</td>
</tr>
<tr>
<td>3</td>
<td>2.10/1.02</td>
<td>1.82/0.94</td>
<td>1.74/0.91</td>
<td>1.93/0.99</td>
</tr>
<tr>
<td>4</td>
<td>3.27/1.34</td>
<td>2.66/1.18</td>
<td>2.64/1.14</td>
<td>2.77/1.23</td>
</tr>
<tr>
<td>5</td>
<td>4.27/1.63</td>
<td>3.46/1.45</td>
<td>3.32/1.38</td>
<td>3.51/1.52</td>
</tr>
<tr>
<td>6</td>
<td>5.74/2.10</td>
<td>4.44/1.79</td>
<td>4.16/1.65</td>
<td>4.29/1.80</td>
</tr>
<tr>
<td>7</td>
<td>7.68/2.62</td>
<td>5.82/1.96</td>
<td>5.37/1.84</td>
<td>5.42/1.93</td>
</tr>
</tbody>
</table>
Results

Attention weights predicted by the graph attention mechanism. (a): Lane keeping case; (b): Lane changing case
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Short-term Trajectory Prediction

Trajectory Planning and Tracking
Integration of traffic decisions (strategic-level) and vehicle control (tactical-level)

- Cooperative trajectory planning
- Control Strategy and Speed Planning
- Vehicle kinematics model and dynamical model
Vehicle Models: vehicle kinematics model & vehicle dynamical model

State variables $\xi_{kin} = [X_r, Y_r, \phi]^T$
Control variables $u_{kin} = [v_r, \delta_f]^T$

State-space expression $\dot{\xi}(t) = f_{\mu(t)}^{2\omega}(\xi(t), u(t))$
State variables $\xi_{kin} = [X_r, Y_r, \phi]^T$
Control variables $u_{kin} = [v_r, \delta_f]^T$

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & \tan \delta_f / l \end{bmatrix} \begin{bmatrix} v_r \end{bmatrix}$$

$$m\ddot{x} = m\dot{y}\dot{\phi} + F_{xf,l} + F_{xf,r} + F_{xr,l} + F_{xr,r}$$
$$m\ddot{y} = -m\dot{x}\dot{\phi} + F_{yf,l} + F_{yf,r} + F_{yr,l} + F_{yr,r}$$
$$I\ddot{\phi} = a(F_{yf,l} + F_{yf,r}) - b(F_{yr,l} + F_{yr,r}) + c(-F_{xf,l} + F_{xf,r} - F_{xr,l} + F_{xr,r})$$
Cooperative trajectory planning

Mainline

\[ \begin{align*}
  \text{Adjustment} & \quad \text{Trajectory prediction} \\
  \text{(A)} & \quad \text{(B)} \\
  t=t_0 & \quad t=t_f \\
  k_{\text{fol}} & \quad k_{\text{fol}} \\
  k & \quad k \\
  t=t_0 & \quad t=t_f \\
  k_{\text{fol}} & \quad k_{\text{fol}}
\end{align*} \]

Ramp

\[ \begin{align*}
  \text{Adjustment} & \quad \text{Trajectory planning} \\
  t=t_0 & \quad t=t_f \\
  k & \quad k \\
  t=t_0 & \quad t=t_f \\
  k & \quad k
\end{align*} \]

Cost Function

\[ w_1(t_f - t_0) \quad \rightarrow \quad \text{Efficiency} \]
\[ w_2(x_k(t_f)) \quad \rightarrow \quad \text{Comfort} & \quad \text{Smoothness} \]
\[ w_3 \int_{t_0}^{t_f} \omega_k^2(t) \quad \rightarrow \quad \text{Cooperativity} \]
\[ w_4 \int_{t_0}^{t_f} j_{kx}^2(t) dt + \int_{t_0}^{t_f} j_{ky}^2(t) dt \]
\[ w_5 \]

Based on CDMMT
Results—trajectory planning

merging trajectory with a constant speed of 24 m/s

merging trajectory under different speed
Trajectory Tracking

Simulation results of trajectory tracking under constant speed
Trajectory Tracking

Speed profile generation (Optimality Theory)

Based on a given trajectory \( S \)

\[
\begin{align*}
\omega_1 \int_{t_0}^{t_n} (S'')^2 dt & \quad \text{Measures of the speed profile smoothness (comfort)} \\
\omega_2 \int_{t_0}^{t_n} (S''')^2 dt & \quad \text{Measures of the trajectory tracking error (MPC)} \\
\omega_3 \int_{t_0}^{t_n} (S - S_{\text{ref}})^2 dt & \quad \text{Measures of the trajectory tracking efficiency} \\
\omega_4 \int_{t_0}^{t_n} (V_{\text{max}} - V_t)^2 dt & \\
\end{align*}
\]

Cost Function

limiting-velocity of road \( V_t \) (Known)

limiting-velocity according to the curvature of the road \( V_c \)

\[
V_c = \sqrt{\frac{DgR}{2H}}
\]

\( V_{\text{max}} \leq \min(V_t, V_c) \)

D: distance between rear wheels
R: Turning radius
H: Height of car body center of gravity to ground
Trajectory Tracking——Current work

1. Solving the optimal model of speed profile generation
   Method: Dynamic programming/Quadratic programming/QP+DP
2. Designing the speed tracking controller based on PID
3. Designing the longitudinal and lateral coupling controller
   (Tracking a given trajectory with a desired speed profile)
4. Evaluating the tracking performance of the designed controller according to the joint simulation results
Future directions

Integration of trajectory planning and trajectory tracking (considering trajectory re-planning)

Time-delay systems

Game theoretical approach for microscopic right-of-way trading considering heterogeneous users and bounded rationality

Stochasticity in the problem (HV, CAV)

Generalization: e.g., intersections

Road test experiments


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